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INVESTIGATION OF THE SCATTERING OF HARMONIC ELASTIC WAVES BY TWO COLLINEAR SYMMETRIC CRACKS USING THE NON-LOCAL THEORY *

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Abstract: *The scattering of harmonic waves by two collinear symmetric cracks is studied using the non-local theory. A one-dimensional non-local kernel was used to replace a two-dimensional one for the dynamic problem to obtain the stress occurring at the crack tips. The Fourier transform was applied and a mixed boundary value problem was formulated. Then a set of triple integral equations was solved by using Schmidt's method. This method is more exact and more reasonable than Eringen's for solving this problem. Contrary to the classical elasticity solution, it is found that no stress singularity is present at the crack tip. The non-local dynamic elastic solutions yield a finite hoop stress at the crack tip, thus allowing for a fracture criterion based on the maximum dynamic stress hypothesis. The finite hoop stress at the crack tip depends on the crack length, the lattice parameter and the circular frequency of incident wave.*

Key words: the non-local theory; Schmidt's method; the triple-integral equation
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Introduction

The last four decades have witnessed the inauguration of a novel theory of material bodies, named the non-local mechanics. This was done primarily due to the efforts of Edelen[1], Eringen[2], Green and Rivlin[3]. According to the non-local theory, the stress at a point X in a body depends not only on the strain at point X but also on those at all other points of the body. This is different from the classical theory. For the classical theory, the stress at a point X in a body depends only on the strain at point X. In Eringen's papers [4 – 7], the state of stress near the tip of a sharp line crack in an elastic plate subjected to uniform tension, in-plane shear and anti-plane shear are discussed. The field equations employed in the solutions of these problems

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are those of the theory of the non-local elasticity. The solutions gave finite stress at the crack tips, thus resolving a fundamental problem that has remained unsolved over half a century. This enabled us to employ the maximum-stress hypothesis to deal with fracture problem and the composite materials problem in a natural way. However, they were not exact and there is oscillatory stress near the crack tip[4]. The iteration error is also not reasonable[4-7], because the dual integral equation has a super singularity integral kernel,. To overcome the difficulty, Schmidt's method[8] will be used. In recent papers [9 - 13], the static state and the dynamic state problems for a crack were investigated by using the non-local theory with Schmidt's method, respectively. To the author's knowledge, analytical treatment of two transient I-cracks problem by using the non-local theory has not been attempted.

For the above-mentioned reasons, the present paper deals with the dynamic problem of two collinear cracks in an elastic plate by using the non-local theory. The field equations of non-local elasticity theory was employed to formulate and solve this problem. The Fourier transform is applied and a mixed boundary value problem is formulated. Then a set of triple integral equations is solved with a new method, namely, Schmidt's method[8]. In solving the equations, the crack surface displacement is expanded in a series using Jacobi's polynomials and Schmidt's method is used. This process is quite different from that adopted in Eringen's papers [4 - 7] and can overcome mathematical difficulty involved. The solution in this paper is more accurate and more reasonable than Eringen's ones. The solution, as expected, does not contain the stress singularity near the crack tips. The stress along the crack's line depends not only on the crack length, but also on the lattice parameter and the circular frequency of incident wave. However, the stress resulting from the classical theory depends only on the crack length.

1 Basic Equations of Non-local Elasticity

According to the non-local theory, the stress at a point X in a body depends not only on the strain at point X but also on those at all other points of the body. This observation is in accordance with atomic theory of the lattice dynamics and experimental observation of the phonon dispersion[14]. Basic equations of linear, homogeneous, isotropic, non-local elastic solids, with vanishing body force are

$$\tau_{kl,k} = \rho \ddot{u}_l, \tag{1}$$

$$\tau_{kl}(X,t) = \int_V \alpha(|X' - X|)\sigma_{kl}(X',t)\mathrm{d}V(X'), \tag{2}$$

where

$$\sigma_{ij}(X',t) = \lambda u_{r,r}(X',t)\delta_{ij} + \mu[u_{i,j}(X',t) + u_{j,i}(X',t)], \tag{3}$$

where the only difference from classical elasticity is in the stress constitutive equation (2) in which the stress $\tau_{kl}(X)$ at a point X depends on the strains $e_{kl}(X')$, at all points of the body. For homogeneous and isotropic solids there exist only two material constants, λ and μ are the Lame constants of classical elasticity. ρ is the mass density of the material. $\alpha(|X' - X|)$ is known as influence function, and is the function of the distance $|X' - X|$. The expression (3) is the classical Hook's law. Substitution of equation (3) into equation (2) and using Green-Gauss theorem, we can obtain

$$\int_V \alpha(|X' - X|)[(\lambda + \mu)u'_{k,kl}(X',t) + \mu u'_{l,kk}(X',t)]\mathrm{d}V(X') -$$

$$\int_{\partial V}\alpha(|X'-X|)\sigma_{kl}(X',t)\mathrm{d}a_k(X')=\rho\ddot{u}_l. \tag{4}$$

Here the surface integral may be dropped if the only surface of the body is at infinity.

2 The Crack Model

It is assumed that there are two collinear symmetric cracks of length $1-b$ along the x-axis in an elastic plate as shown in Fig.1. $2b$ is the distance between the two cracks. Let ω be the circular frequency of the incident wave. $-\tau_0$ is a magnitude of the incident wave. In what follows, the time dependence of all field quantities assumed to be of the form $e^{-i\omega t}$ will be suppressed but understood. It was further supposed that the two faces of the crack do not come in contact during vibrations. The solution of two collinear symmetric cracks of arbitrary finite length can easily be obtained by a simple change in the numerical values of the present problem. When the cracks are subjected to the harmonic elastic waves, as discussed in [15], the boundary conditions on the crack faces at $y=0$ are (b is a dimensionless variable):

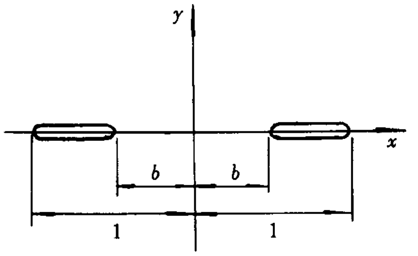

Fig.1 Incidence of a time harmonic wave on two collinear symmetric cracks of the length $1-b$

$$\tau_{yx}(x,0,t)=0,\quad v(x,0,t)=0,\quad |x|>1,\ |x|<b, \tag{5}$$

$$\tau_{yx}(x,0,t)=0,\quad \tau_{yy}(x,0,t)=-\tau_0,\quad b\leqslant|x|\leqslant 1, \tag{6}$$

$$u(x,y,t)=v(x,y,t)=0,\quad (x^2+y^2)^{1/2}\to\infty. \tag{7}$$

In this paper, the wave is vertically incident and we only consider that τ_0 is positive.

3 The Triple Integral Equations

According to the boundary conditions, the equation (4) can be written as follows:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\alpha(|x'-x|,|y'-y|)[(\lambda+\mu)u'_{k,kj}(x',y',t)+\mu u'_{j,kk}(x',y',t)]\mathrm{d}x'\mathrm{d}y'-2\mu\left\{\int_{-1}^{-b}+\int_{b}^{1}\right\}\alpha(|x'-x|,|y|)\times[e_{2j}(x',0,t)]\mathrm{d}x'=-\rho\omega^2u_j, \tag{8}$$

where $[e_{2j}(x',0,t)]=e_{2j}(x',0^+,t)-e_{2j}(x',0^-,t)$ is a jump across the crack.

$$e_{kj}(x,y,t)=0.5[u_{k,j}(x,y,t)+u_{j,k}(x,y,t)].$$

From the reference [7], it can be obtained:

$$[e_{2j}(x,0,t)]=0\qquad\text{for all } x. \tag{9}$$

Define the Fourier transform by the equations

$$\bar{f}(s)=\int_{-\infty}^{\infty}f(x)e^{-isx}\mathrm{d}x, \tag{10}$$

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{f}(s)e^{isx}\mathrm{d}s. \tag{11}$$

For solving the problem, the Fourier transform of equation (8) with respect x can be given as follows:

$$\int_{-\infty}^{\infty} \bar{\alpha}(|s|, |y' - y|)[\mu\bar{u}'_{,yy} - (\lambda + 2\mu)s^2\bar{u}' - is(\lambda + \mu)\bar{v}'_{,y}]dy' = -\rho\omega^2\bar{u}, \tag{12}$$

$$\int_{-\infty}^{\infty} \bar{\alpha}(|s|, |y' - y|)[-is(\lambda + \mu)\bar{u}'_{,y} + (\lambda + 2\mu)\bar{v}'_{,yy} - s^2\mu\bar{v}']dy' = -\rho\omega^2\bar{v}. \tag{13}$$

For the influence function α, as discussed in [9, 11, 13, 16, 17], it seems obvious that one has to resort to an approximate procedure. In the given problem, the appropriate numerical procedure seems to spring quite naturally from the hypothesis of the attenuating neigborhood underlying the theory of the non-local continua. According to this hypothesis, the influence of the particle of the body, on the thermoelectric state at the particle under observation, subside rather rapidly with an increasing distance between the two particles. In classical theory, the function that characterizes the particle interactions is the Dirac delta function since in this theory the actions are assumed to have a zero range. In non-local theories the intermolecular forces may be represented by a variety of functions as long as their values decrease rapidly with the distance. In present study, as adequate functions, it was decided to select the terms:

$$\bar{\alpha}(|s|, |y' - y|) = \bar{\alpha}_0(s)\delta(y' - y). \tag{14}$$

From equations (12) and (13), it can be gotten

$$\bar{\alpha}_0(s)[\mu\bar{u}_{,yy} - (\lambda + 2\mu)s^2\bar{u} - is(\lambda + \mu)\bar{v}_{,y}] = -\rho\omega^2\bar{u}, \tag{15}$$

$$\bar{\alpha}_0(s)[-is(\lambda + \mu)\bar{u}_{,y} + (\lambda + 2\mu)\bar{v}_{,yy} - s^2\mu\bar{v}] = -\rho\omega^2\bar{v}, \tag{16}$$

whose solutions do not present difficulties, it can be obtained ($y \geqslant 0$)

$$u(x, y, t) = -\frac{2}{\pi}\int_0^{\infty} sA_1(s)\sin(sx)\exp(-\gamma_1 y)ds - \frac{2}{\pi}\int_0^{\infty} \gamma_2 A_2(s)\sin(sx)\exp(-\gamma_2 y)ds, \tag{17}$$

$$v(x, y, t) = -\frac{2}{\pi}\int_0^{\infty} \gamma_1 A_1(s)\cos(sx)\exp(-\gamma_1 y)ds - \frac{2}{\pi}\int_0^{\infty} sA_2(s)\cos(sx)\exp(-\gamma_2 y)ds, \tag{18}$$

where

$$\gamma_1^2 = s^2 - \frac{\omega^2}{c_1^2\bar{\alpha}_0(s)}, \quad \gamma_2^2 = s^2 - \frac{\omega^2}{c_2^2\bar{\alpha}_0(s)},$$

$$c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad c_2 = \sqrt{\frac{\mu}{\rho}}.$$

Now, let the function $A(s)$ be defined such that

$$A_1(s) = -\frac{1}{2\gamma_1}[s^2 + \gamma_2^2]\bar{\alpha}_0(s)A(s), \tag{19}$$

$$A_2(s) = s\bar{\alpha}_0(s)A(s) \tag{20}$$

because of symmetry, it suffices to consider the problem in the problem in the first quadrant only. The boundary conditions can be applied to yield

$$\int_0^{\infty} A(s)\cos(sx)ds = 0 \quad (0 < x < b, \ x > 1), \tag{21}$$

$$\int_0^\infty \bar{\alpha}_0^2(s)f(s)A(s)\cos(sx)\mathrm{d}s = \frac{\pi\tau_0}{2\mu} \quad (b < x < 1). \tag{22}$$

The equations (21) and (22) are the triple integral equations of this problem. In equation (22), $f(s)$ is given as follows:

$$f(s) = \frac{1}{2\gamma_1}\{[s^2 + \gamma_2^2]^2 - 4s^2\gamma_1\gamma_2\}. \tag{23}$$

4 Solution of the Triple Integral Equation

The only difference between the classical and non-local equations is in the introduction of the function $\bar{\alpha}_0(s)$, it is logical to utilize the classical solution to convert the system equations (21) and (22) to an integral equation of the second kind which is generally better behaved. For $a = 0$, then $\bar{\alpha}_0(s) = 1$ and equations (21) and (22) reduce to the triple integral equations for the same problem in classical elasticity. Of course, the triple integral equations can be considered to be a single integral equation of the first kind with a discontinuous kernel[4]. It is well-known in the literature that integral equations of the first kind are generally ill-posed in sense of Hadamard, i.e. small perturbations of the data can yield arbitrarily large changes in the solution. This makes the numerical solution of such equations quite difficult. In this paper, the Schmidt's method was used to overcome the difficulty. As is discussed in [7] and [17], it was taken

$$\alpha_0 = \chi_0 \exp(-(\beta/a)^2(x' - x)^2), \tag{24}$$

$$\chi_0 = \frac{1}{\sqrt{\pi}}\beta/a, \tag{25}$$

where β is a constant; a is the lattice parameter.
So we obtain

$$\bar{\alpha}_0(s) = \exp\left(-\frac{(sa)^2}{(2\beta)^2}\right). \tag{26}$$

$\bar{\alpha}_0(s) = 1$ for the limit $a \to 0$, so that the equation (22) reduces to the well-known equation of the classical theory. Here the Schmidt method[8] can be used to solve the triple integral equations (21) and (22). The displacement v was represented by the following series:

$$v = \sum_{n=0}^\infty a_n P_n^{(1/2,1/2)}\left(\frac{x - \frac{1+b}{2}}{\frac{1-b}{2}}\right)\left(1 - \frac{\left(x - \frac{1+b}{2}\right)^2}{\left(\frac{1-b}{2}\right)^2}\right)^{1/2} \quad \text{for } b < x \leqslant 1,\ y = 0, \tag{27}$$

$$v = 0 \qquad (\text{for } x > 1,\ x < b,\ y = 0), \tag{28}$$

where a_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial[18]. The Fourier transformation of equation (27) is[18]:

$$-\frac{\omega^2}{2c_2^2}A(s) = \bar{v}(s,0,t) = \sum_{n=0}^\infty a_n B_n G_n(s)\frac{1}{s}J_{n+1}\left(s\frac{1-b}{2}\right), \tag{29}$$

$$B_n = 2\sqrt{\pi}\,\frac{\Gamma(n+1+\frac{1}{2})}{n!}, \quad G_n(s) = \begin{cases}(-1)^{n/2}\cos\left(s\frac{1+b}{2}\right) & (n = 0,2,4,6,\cdots), \\ (-1)^{n-1/2}\sin\left(s\frac{1+b}{2}\right) & (n = 1,3,5,7,\cdots),\end{cases} \tag{30}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting equation (29) into equations (21) and (22), respectively, the equation (21) has been automatically satisfied, the equation (22) reduces to the form for $b < x < 1$

$$\sum_{n=0}^{\infty} a_n B_n \int_0^{\infty} \bar{\alpha}_0^2(s) G_n(s) f(s) \frac{1}{s} J_{n+1}\left(s \frac{1-b}{2}\right)\cos(xs)\,\mathrm{d}s = -\frac{\pi\tau_0\omega^2}{4\mu c_2^2}. \tag{31}$$

For a large s, the integrands of the equation (31) almost all decrease exponentially. So the semi-infinite integral in equation (31) can be evaluated numerically by Filon's method[19]. Thus equation (31) can be solved for coefficients a_n by the Schmidt's method[8]. For brevity, the equation (31) can be rewritten as

$$\sum_{n=0}^{\infty} a_n E_n(x) = U(x) \quad (b < x < 1), \tag{32}$$

where $E_n(x)$ and $U(x)$ are known functions and coefficients a_n are unknown and will be determined. A set of functions $P_n(x)$ which satisfy the orthogonality condition

$$\int_b^1 P_m(x) P_n(x)\,\mathrm{d}x = N_n \delta_{mn}, \quad N_n = \int_b^1 P_n^2(x)\,\mathrm{d}x \tag{33}$$

can be constructed from the function, $E_n(x)$, such that

$$P_n(x) = \sum_{i=0}^{n} \frac{M_{in}}{M_{nn}} E_i(x), \tag{34}$$

where M_{ij} is the cofactor of the element d_{ij} of D_n, which is defined as

$$\boldsymbol{D}_n = \begin{bmatrix} d_{00}, d_{01}, d_{02}, \cdots, d_{0n} \\ d_{10}, d_{11}, d_{12}, \cdots, d_{1n} \\ d_{20}, d_{21}, d_{22}, \cdots, d_{2n} \\ \cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots \\ d_{n0}, d_{n1}, d_{n2}, \cdots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_b^1 E_i(t) E_j(x)\,\mathrm{d}x. \tag{35}$$

Using equations (32) - (35), we obtain

$$a_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \tag{36}$$

with

$$q_j = \frac{1}{N_j} \int_b^1 U(x) P_j(x)\,\mathrm{d}x. \tag{37}$$

5 Numerical Calculations and Discussion

When coefficients a_n are known, the entire stress field is obtainable. However, in fracture mechanics, it is of importance to determine stress τ_{yy} along the crack line. τ_{yy} at $y = 0$ is given as follows:

$$\tau_{yy} = \frac{4\mu c_2^2}{\pi\omega^2} \sum_{n=0}^{\infty} a_n B_n \int_0^{\infty} \bar{\alpha}_0^2 G_n(s) f(s) \frac{1}{s} J_{n+1}\left(s \frac{1-b}{2}\right) \cos(sx)\,\mathrm{d}s. \tag{38}$$

For $a = 0$ at $x = 1$, b, we have the classical stress singularity. However, so long as $a \neq 0$, the equation (38) gave a finite stress all along $y = 0$. At $b < x < 1$, τ_{yy}/τ_0 is very close to unity, and for $x > 1$, τ_{yy}/τ_0 possesses finite values diminishing from a maximum value at $x = l$ to zero at $x = \infty$. Since $a/[2\beta(1-b)] > 1/100$ represents a crack length of less than 100 atomic distances[6], and such submicroscopic sizes, other serious questions arise regarding the interatomic arrangements and force laws. We do not pursue solutions at such small crack sizes. The dynamic stress is computed numerically for the Lame constants $\lambda = 98 \times 10^9 (\mathrm{N/m^2})$, $\mu = 77 \times 10^9 (\mathrm{N/m^2})$, $\rho = 7.7 \times 10^3 (\mathrm{kg/m^3})$. The semi-infinite numerical integrals, which occur, are evaluated easily by Filon and Simpson' s methods because the rapid diminution of the integrands. From Refs. [20] and [21], it can be seen that the Schmidt's method is performed satisfactorily if the first ten terms of infinite series to equation (31) are retained. The results are plotted in Figs. 2 – 11.

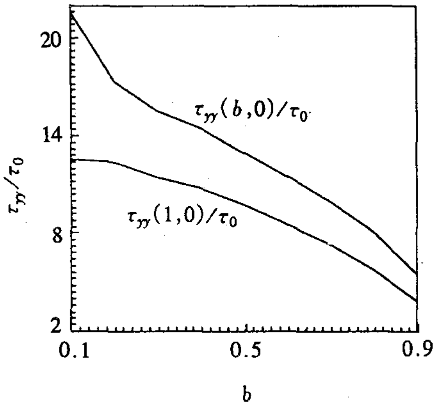

Fig. 2 The variation of the stress at the crack tips for $a/(2\beta) = 0.000\,5$, $\omega/c_2 = 0.2$

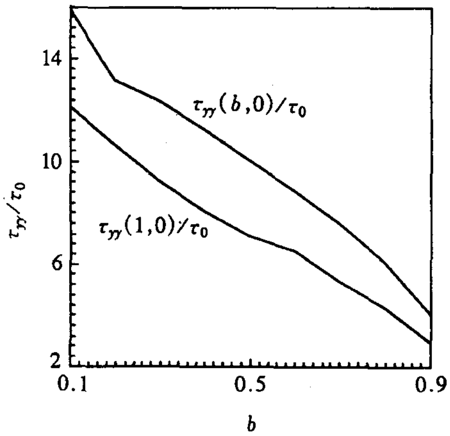

Fig. 3 The variation of the stress at the crack tips for $a/(2\beta) = 0.000\,8$, $\omega/c_2 = 1.0$

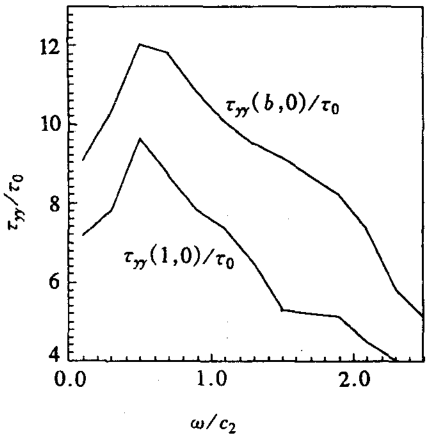

Fig. 4 The variation of the stress at the crack tips for $a/(2\beta) = 0.000\,8$, $b = 0.4$

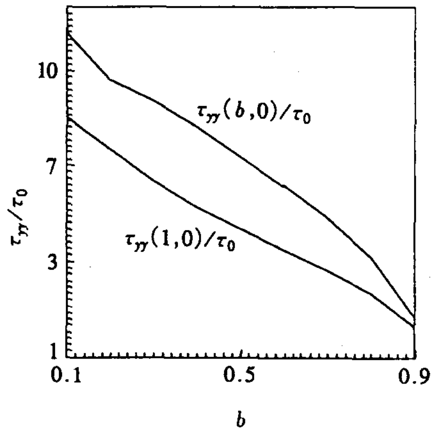

Fig. 5 The variation of the stress at the crack tips for $a/(2\beta) = 0.001\,5$, $\omega/c_2 = 1.0$

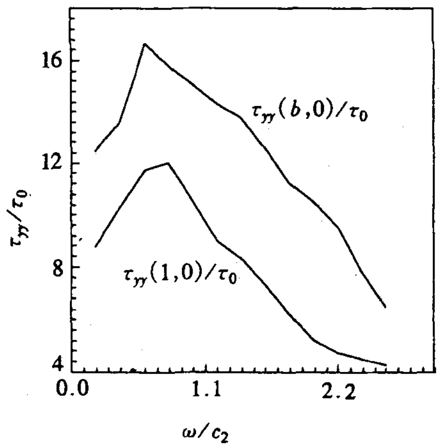

Fig.6 The variation of the stress at the crack tips for $a/(2\beta)$ = 0.000 5, b = 0.4

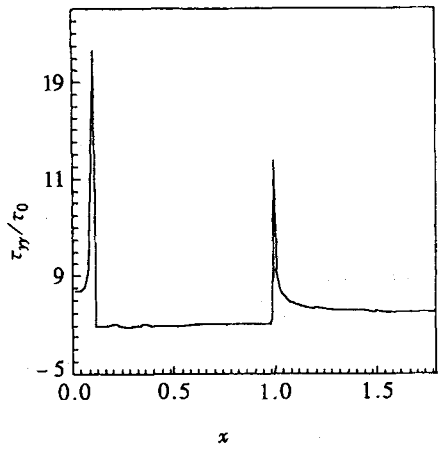

Fig.7 The variation of stress on the crack line for $a/(2\beta)$ = 0.000 5, ω/c_2 = 0.2, b = 0.1

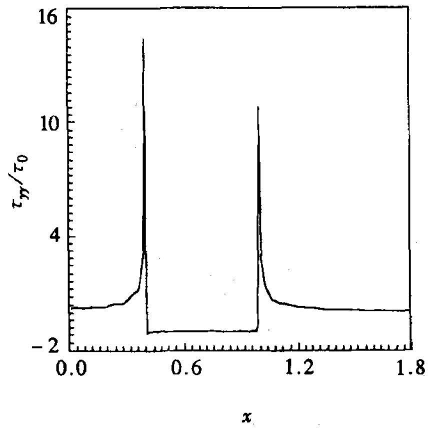

Fig.8 The variation of stress on the crack line for $a/(2\beta)$ = 0.000 5, ω/c_2 = 0.2, b = 0.4

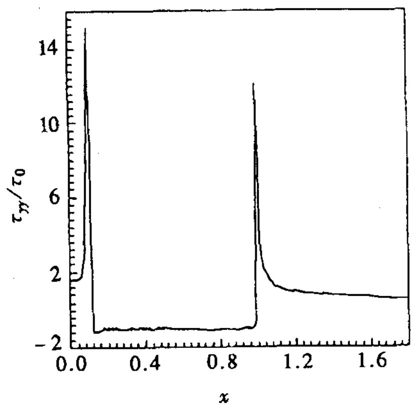

Fig.9 The variation of stress on the crack line for $a/(2\beta)$ = 0.000 8, ω/c_2 = 1.0, b = 0.1

The following observations can be made:

i) The method that used in this paper can overcome the mathematics difficulties that occur in Eringen's papers [4, 6, 7]. The results are more accurate than Eringen's ones. The method is more reasonable than Eringen's ones;

ii) The maximum stress does not occur at the crack tip, but slightly away from it. This phenomenon has been thoroughly substantiated by Eringen[14]. The maximum stress is finite. The distance between the crack tip and the maximum stress point is very small. This distance depends on the lattice parameter, the crack length and the circular frequency of the incident wave.

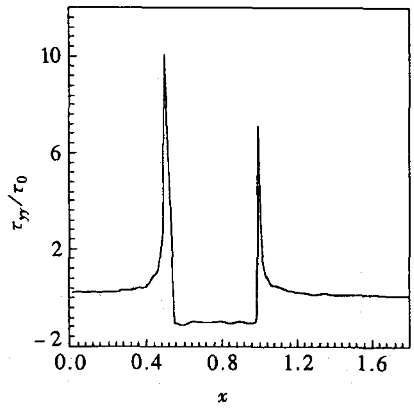

Fig.10 The variation of stress on the crack line for $a/(2\beta) = 0.000\,8$, $\omega/c_2 = 1.0$, $b = 0.5$

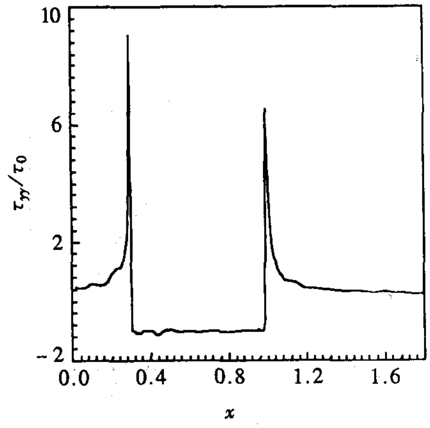

Fig.11 The variation of stress on the crack line for $a/(2\beta) = 0.001\,5$, $\omega/c_2 = 1.0$, $b = 0.3$

Contrary to the classical elasticity solution, it is found that no stress singularity is present at the crack tip, and alsothe present results converge to the classical ones for positions when far away from the crack tip;

iii) The normal stress at the crack tip becomes infinite as the atomic distance $a \to 0$. This is the classical continuum limit of square root singularity;

iv) For the a/β = constant, viz., the atomic distance does not change, the value of the dynamic stress concentrations (at the crack tip) becomes higher with the increase of the crack length. Note this fact, experiments indicate that materials with smaller cracks are more resistant to fracture than those with larger cracks;

v) The variation of the stresses is nonlinear with increasing of the frequency, The stresses do not increase for all the frequency;

vi) The significance of this result is that the fracture criteria are unified at both the macroscopic and microscopic scales;

vii) The left tip's stress is greater than the right tip's stress for the right crack. At the end of the right crack, the stress on the crack line becomes lower with the increasing of the distance between the two cracks.

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